A Method for Optimizing the Weight and Drag Relation of Streamlined Bicycles

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Abstract

The original purpose of this study was to determine if it is possible to find an exact solution to the governing differential equation for motion of a Human-Powered Vehicle on hilly terrain. None was found so further effort was directed at solving the differential equation numerically. Several numerical integration methods were evaluated, with a trapezoidal scheme being selected being selected for its simplicity. Time histories were generated by integrating the equation numerically. The resulting conclusion is that a fairing always results in an increase in speed for hill sizes up to 100m (under the assumptions made in this study). There is no condition where the fairing will cause greater slowing due its weight than the aerodynamic advantage it provides when the cyclist is pedalling continuously. The method for this comparison will be used in future studies, to determine whether the introduction of braking and coasting change these conclusions.

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1. Chapter 1. Introduction and Motivation

Concerns about the environment and traffic congestion have resulted in more people taking to the roads under their own power on bicycles. However, the existing bicycle designs that are available do not provide a solution for someone who wishes to abandon their car completely and in all types of weather. A bike that would be practical for such a person would need to have adequate weather protection. Here in the Pacific Northwet where the worst conditions bring nothing worse than light rain and cool weather a full-fairing is all that is required. Many riders object to the idea of a complete enclosure on the basis that the extra weight would make hill-climbing more difficult. In this rider's experience overall trip speeds were faster with a full-fairing. A systematic method was desired to prove, analytically, when the fairing is worth its weight.

A few fairings for bikes are available today, one of the most popular of which is the Zzipper shown in Figure 1. This can be purchased easily, is made to mount on most recumbents and results in an astonishing performance improvement. Although it covers less than 25% of the bike's length it cuts drag by half, resulting in a speed improvement of 20%. The reason for the popularity of this fairing is its relatively low price, low weight and the fact that it is available on test-ride bikes where a prospective buyer can feel the advantage first hand.

Figure 1-1. Tour Easy with Zzipper fairing covering less than 25% of bike length.

The author has also studied human-powered vehicles with more extensive fairings and built a fully-faired vehicle which was used for year-round commuting in the Seattle area (See http://www.mcs.net/~gkpsol/coolbikes.html for these and other homebuilt streamlined bikes). Based on this knowledge it became clear that even a fully-faired recumbent (with its attendant weight increase) on hilly terrain permitted faster commuting times than an unfaired vehicle. Such bikes are not commercially available, however, and so an analytical method must be available to prove the fact. In this paper a method is developed to quantify that advantage.

Chapter 2. Theory

The governing differential equation as derived in **HEI99** is:

$$n_x = x''/g = \eta*(P/W)/x' - (\mu + \gamma) - [.5*\rho / (W/CdA)] * (x')^2$$

n_x	longitudinal acceleration as a fraction of gravity
x''	acceleration
x'	speed
x	position along the path
P	power input by the rider
W	weight of the vehicle and rider
μ	rolling resistance coefficient
γ	slope of the terrain
η	mechanical efficiency of the drivetrain.
ρ	atmospheric air density
CdA	effective frontal area of the vehicle

This equation is integrated to determine the position of the vehicle as a function of time, and ultimately the time taken to complete a given course with specified slope (γ_{max}) and height of the terrain (h_{max}).

Note that x''/g is the acceleration of the vehicle as a fraction of gravitational acceleration, it will be referred to as $\mathbf{n_x}$. From the equation, it is analagous to slope. That is if no other forces are acting on the vehicle as it rolls down an incline of .05 or 5%, its acceleration will be .05 g's. Similarly the other terms can be considered as "equivalent" slopes or acceleration. A rolling coefficient of .006 will result in a decceleration of .006 g's on level ground, or a reduction in climbing capability of .6 % grade.

The power-to-weight ratio (P/W) of the vehicle/rider combination determines the maximum climbing capability. Its units are m/s, meaning the maximum speed at which it can move vertically in the absence of any other forces. Dividing it by the forward velocity yields the maximum acceleration capability on level ground at that velocity, or the maximum possible climbing angle at that velocity.

The weight-to-drag area (W/CdA) ratio of the vehicle/rider combination is a measure of its aerodynamic efficiency. This ratio is exactly analagous to the L/D ratio used as a measure of aircraft efficiency. In metric units this number is very large, because it reflects an W/D at 1 Pascal of dynamic pressure (corresponding to a speed of .01 m/s). For this paper, the value is being quoted at 10 m/s (expressed as WD-10) to give a more meaningful number. In physical terms WD-10 is the slope (meters forward per meter of altitude drop) on which the vehicle can maintain a constant speed (zero acceleration) with no other forces acting.

The slope of terrain γ is in reality an arbitrary function of where you are on Earth. However, for this study sinusoidal hills were used to permit a systematic variation of terrain. Realistic hills have been observed to have single- or multi-modal distributions but were considered to be beyond the scope of this study.

Chapter 3. Variation of Input Parameters

Vehicle Parameters

Despite the fact that this analysis scheme uses mostly non-dimensional input parameters, it was necessary to set those inputs at reasonable numerical values so a single combination of rider weight, vehicle weight and frontal area (**A**) was used as the basis for determining the baseline condition. From this baseline the percentage of fairing coverage was varied from 0% to 100% of the bike's length and the effective frontal area (**CdA**) was calculated based on data from **MAL83**. Rolling resistance (μ) and mechanical efficiency η were left constant throughout this study. Rolling resistance was taken from **LAF00** to be .006 and mechanical efficiency was taken from **SPI00** as 95%. Other assumed values are presented in Figure 3.1.

Parameter	Value	Units
Rider Power	200	Watts
Rider Wt.	70	kg
Frontal Area	.60	m^2

Figure 3-1. Baseline Values for Input Parameters

Power to weight (**P/W**) ratio is calculated as the rider input power divided by the weight of the rider/vehicle combination. Naturally, weight will vary with the weight of the fairing and so assumptions were made about the fairing weight. These are presented in Figure 3-2. It is worth noting that **P/W** units of m/s and represents the maximum possible vertical speed that can be accomplished. Vertical speed will always be maximum for a bare rider running up a hill. However, since the purpose of a vehicle is to permit the fastest forward speed, it is always beneficial to improve the efficiency of forward speed using such devices as wheels, gear trains and fairings.

The amount of weight added by a fairing depends on the details of its construction. The fairing was assumed to be an ellipsoidal shell with the characteristics described in Figure 3.2 and shown in sideview in Figure 3.4.

Fairing	Width	Length	Height	Thickness	
Dimensions	0.5	2.66	1.2	0.005	
Volume	0.11	m^3			
Density	73.64	kg/m^3			
Weight	7.90	kg			
Support Structure	7.90	kg			
Total Fairing	15.80	kg			
Bike Frame	15	kg			
Coverage (%)	**100%**	**75%**	**50%**	**25%**	**0%**
Total Veh. wt	31	27	23	19	15

Figure 3-2. Fairing Characteristics Used for Calculating Vehicle Weight

These numbers for weight and geometry are based primarily on the author's experience with various fully-faired and partially faired HPV's. It is also necessary to determine (or estimate) the **CdA** for each value of fairing coverage. These values were estimated based on data in **MAL83** and are shown in Figure 3.3.

Coverage	CdA	Reduction	W/CdA	W/D-10	P/W
0%	0.48	0.0%	1737	28	0.2399
25%	0.258	46.3%	3382	55	0.2292
50%	0.234	51.3%	3895	63	0.2195
75%	0.144	70.0%	6598	107	0.2105
100%	0.072	85.0%	13734	223	0.2023

Figure 3-3. Input Parameters for Various Values of Fairing Coverage

As discussed in the previous section, the parameter **W/D-10** (W/D at 10 m/s) was defined as the actual program input. This is exactly analogous to the L/D ratio used as a measure of performance for aircraft.

It should be noted that these values for fairing coverage assume that the fairing is first applied to the front and then extended progressively further back. Although the fairing is symmetrical, the order that the fairing is applied (starting at the front or starting at the rear) makes a significant difference. Applying the first 25% of fairing to the rear results in less of an improvement than applying it to the front because the front contributes more drag on a typical unfaired vehicle.

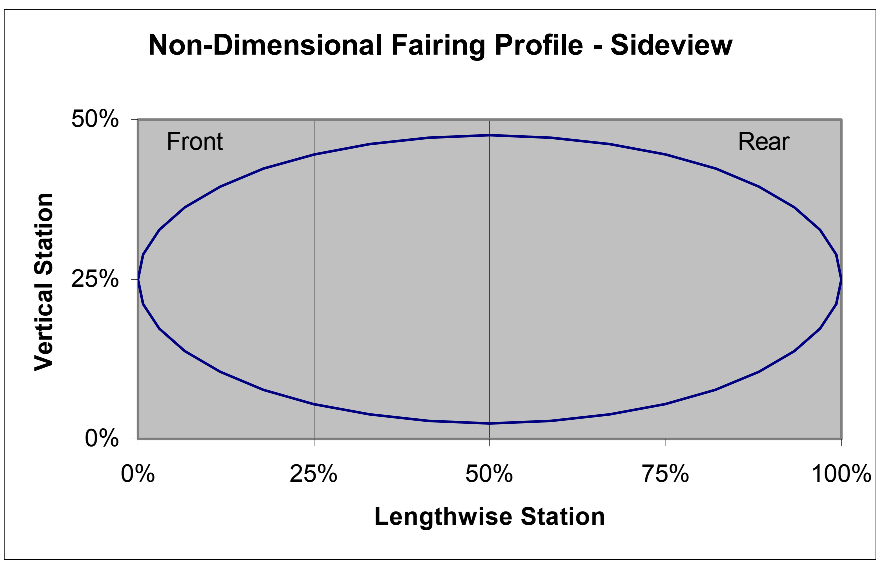

Figure 3-4 Non-Dimensional Profile for Ellipsoidal Fairing

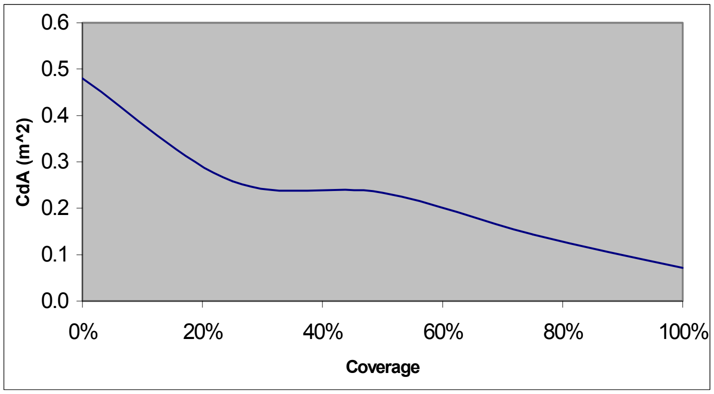

Figure 3-5. Effective Area (CdA) Change with Fairing Coverage

The computer program was used to determine the level-ground speed at constant input power for each fairing configuration. The results are shown below. The purpose of this study is to see if the more faired vehicle still retains a speed advantage when travelling on hilly terrain.

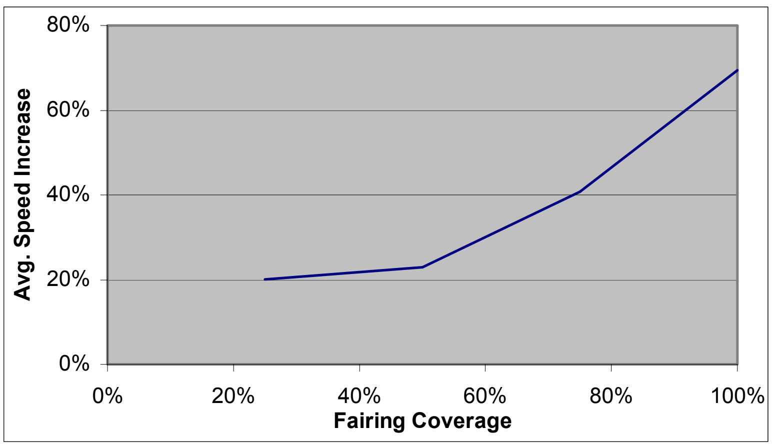

Figure 3-6. Speed Increase with Fairing Coverage

Terrain Parameters

For the purposes of this study, a sinusoidal hill profile was assumed. In the acceleration equation, an expression is required for hill slope (γ) as a function of distance. The expression is:

$$\gamma = \gamma_{max} * \sin(2\pi * x / l_{hill}) \quad \text{or} \quad \gamma = \gamma_{max} * \sin(2x * \gamma_{max} / h_{max})$$

where:

$l_{hill} = \pi * h_{max} / \gamma_{max}$
γ_{max} = the maximum slope on the hill
l_{hill} = the length of the hill for one complete cycle.
h_{max} = the maximum height of the hill

In the first series of test runs γ_{max} is fixed and h_{max} is varied to show its effects. In the second portion, h_{max} is fixed and a range of values are used for γ_{max} to determine if there is a natural frequency for each combination of P/W and W/D. Since real roads have a maximum limit on their slope (assumed to be 10% for this study), γ_{max} was varied from 1-10% in 1% increments.

Chapter 4. Analysis

The equations were converted into a C-language computer program to calculate time history responses. Included in the program was an algorithm for "trimming" or starting the vehicle at a steady state. This made it possible to stabilize the speed before starting each run. The author had previously used trapezoidal integration for the time-domain solution because of its simplicity. There was some concern about the accuracy of this method in this application. A comparison was made to rectangular and Simpson's rule integrations within this program, with less than a 1% difference in the results so the trapezoidal rule was retained. A companion model was created in Simulink/MATLAB to compare with Runge-Kutta integration but results were not available in time for publication. As long as all cases are run with the same method it is expected that the conclusions will be the same with any reasonable integration method.

Each test case was started at a trimmed speed on level ground and then run across 4 hills, to stabilize in a periodic oscillation, before recording data. The time to travel over one complete hill was recorded. The behavior of a vehicle on a sinusoidal hill is like that of a damped oscillator. The aerodynamic drag acts as the damping term (being the only term that is a function of velocity). Several series of tests were run to show the following effects:

1) The effect of increasing fairing coverage (reducing drag), with constant hill size.
It is expected that increasing fairing coverage will reduce the drag and thus increase the expected speeds of the vehicle on level and downhills. When climbing a hill, however, the weight of the fairing will reduce the climbing speed and thus might increase the total time required to complete a hill.

2) The effect of increasing hill size, with constant fairing coverage.
On larger hills a proportionately larger amount of time is spent climbing. If a faired vehicle is heavier because of the weight of a fairing it may be going at a slower speed for a longer time, increasing the total time to complete the hill.

3) The effect of varying gradient with constant hill size.
On hills with greater length (shallower slope) more time is also spent on the climbing portion. Tests were established to determine at what point, if any, a fairing reduces the total time required to complete a hill.

Chapter 5. Results

Effect of Increased Fairing Coverage

For this case a hill with a height of 10m and a maximum grade of 5% was chosen. This is a relatively small rolling hill.

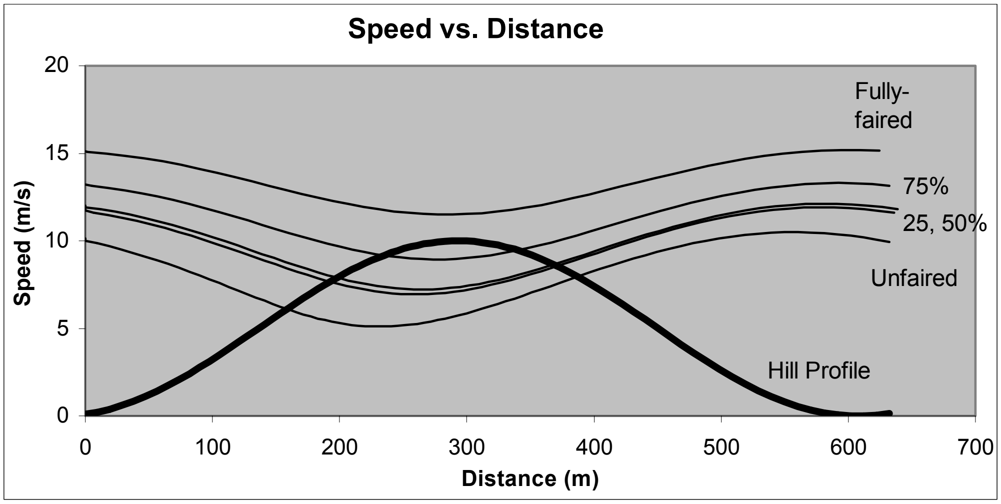

Figure 5-1. Increasing Fairing Coverage on a Constant-sized Hill

It is immediately evident that on such a hill the fairing results in an increased speed on all portions of the hill. There is no point where the higher weight of the faired vehicle would slow it down as much as the increased drag without a fairing would. This is confirmed by the data in Figure 5-2 which show the elapsed time to complete a hill for the same cases. This curve shows a similar trend to the Effective Area curve in Figure 3-5.

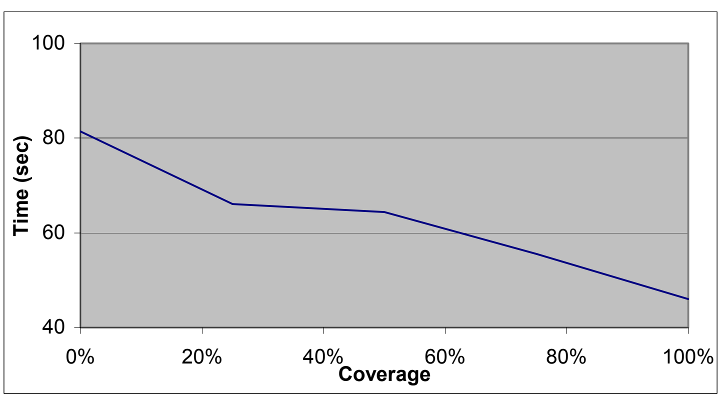

Figure 5-2. Elapsed Time to Complete a Hill with Increasing Fairing Coverage

On small hills such as this a faired vehicle will carry more of its momentum from the downhill portion of the hill to the climb portion. On larger hills this would not be the case, however. Eventually a point will be reached where the momentum runs out and climb rate depends only on P/W. This will typically be at a low speed where there is little benefit from lower drag.

For the purposes of this study it was beneficial to plot the outputs on a phase diagram of speed vs. accelleration. The data of Figure 5-1 are replotted as a phase plot in Figure 5-3. This plot format has the benefit of showing both speed and acceleration on one page and of showing a closed-loop over a complete hill cycle. Acceleration is plotted in g's (fraction of gravity) which are analogous to hill slope. On the steepest part of the slope (.05 grade) a vehicle with no drag and no input power from the rider would accelerate at .05g while descending the hill (top point of the phase loop). The fully-faired vehicle actually reaches this value, indicating that the rider power is approximately equal to the aerodynamic drag. The unfaired vehicle reaches an acceleration of .04g, indicating that the aerodynamic drag is greater than the rider power.

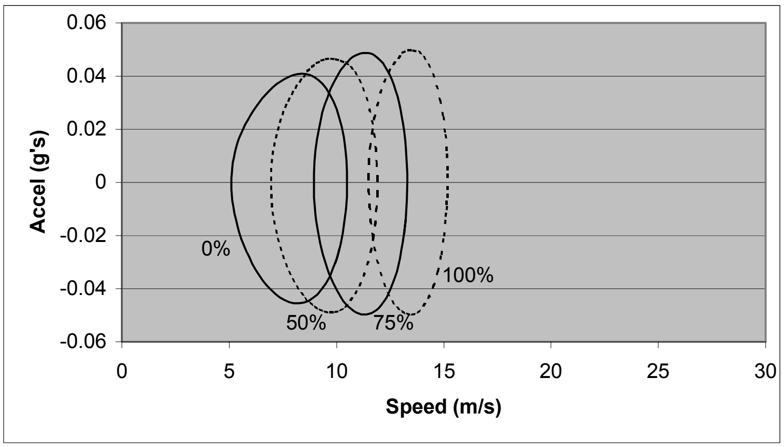

Figure 5-3. Phase Plot Showing Effect of Fairing Coverage (10m Hill – 5% Grade)

On the uphill portion of the hill (lower segment of the phase loop), it would be expected that the decceleration would be -.05g's. On the unfaired vehicle the rider input power is slightly greater than aerodynamic drag and so the acceleration is less negative than -.05. Note, however, in all cases that the speed of the fully-faired vehicle is always greater than that of the unfaired vehicle, even at its slowest climbing speed. This indicates a substantial benefit from a fairing on small hills. The time required to complete the hill with a full fairing is cut nearly in half.

Effect of Increased Hill Size

On larger hills more time is spent climbing, and much of that occurs after the vehicle's momentum from the downhill segment has run out. The phase plot shows this steady climbing segment as the point at the left end of the phase loop. As the hills get larger this portion of the loop goes from being a continuous curve, to a sharp point. For the unfaired vehicle Figure 5-4 exhibits this effect on the 50 and 100m hills. It is also clear that as the hills get larger the maximum and minimum speeds get more extreme, although acceleration is decreased because the second derivative of the hill curve decreases. On the unfaired vehicle this reduction in acceleration is fairly large.

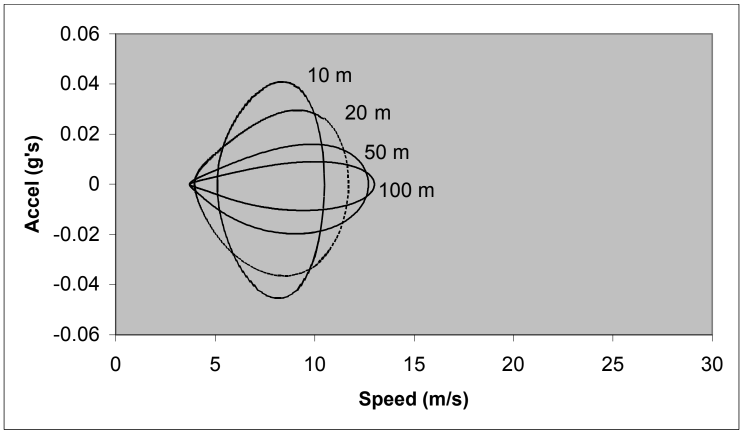

Figure 5-4. Effect of Increased Hill Size on an Unfaired Vehicle

The same cases were run for the faired vehicle and are shown in Fig. 5-5. The point where all of the momentum is used up is higher on the hill with a full fairing. Only the 100m hill case has the sharp point. This is because the effect of reduced aerodynamic drag is a better ability to retain momentum. Additionally, the acceleration on the steepest part of the hill does not decrease much with hill height. Most significantly, the speeds that can be reached on a fully-faired vehicle are much higher than typical cycling speeds. For this reason it is extremely important that such bikes be equipped with adequate brakes. The combination of faster acceleration and higher speeds also makes it important that the rider be aware of his speed at all times because speed climbs rapidly at these accelerations.

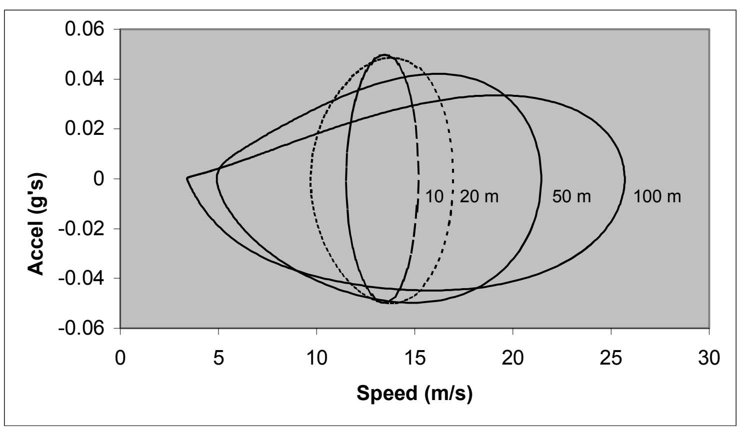

Figure 5-5. Effect of Hill Height on Faired Vehicle Performance

It is apparent from Figure 5-5 that the faired vehicle reaches higher maximum speeds, and so it is expected that the average speed will be higher. However, the main topic of interest was how fast each configuration was relative to the unfaired vehicle – i.e. increase or decrease in average speed. To compare time required to complete a given hill, a series of runs were made varying fairing coverage and hill height. A separate plot was made for each hill height showing the speed increase with varying fairing coverage. These are plotted in Figures 5-6 through 5-9.

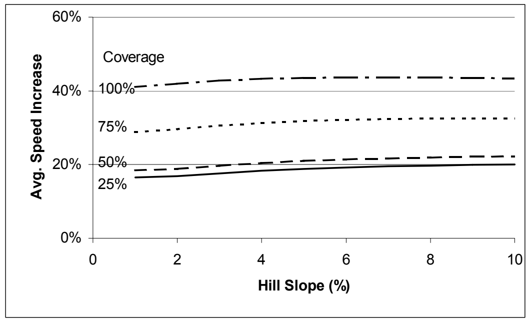

Figure 5-6. Speed Increase due to Fairing Coverage – 10 m Hill

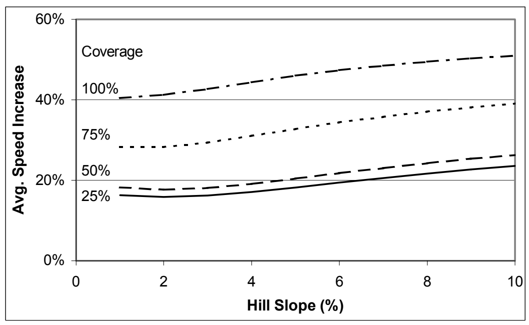

Figure 5-7. Speed Increase due to Fairing Coverage – 20 m Hill

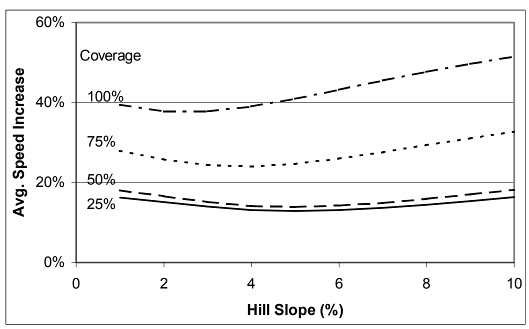

Figure 5-8. Speed Increase due to Fairing Coverage – 50 m Hill

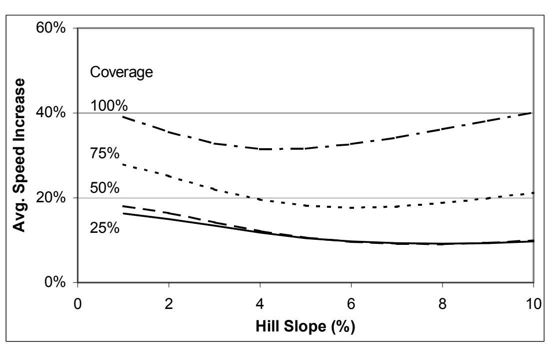

Figure 5-9. Speed Increase due to Fairing Coverage – 100 m Hill

In Figure 5-6 all values of fairing coverage provide an equal speed advantage over the range of hill slope. As the size of the hill increases, some variation with slope appears. It seems as though a minimum value of speed advantage exists, which moves to a different location depending on the size of the hill. At 10m there is a barely-detectable minimum which appears to be between zero and one percent slope. As hill size increases this minimum moves to the right for all values of fairing coverage. However, it moves to the right faster for the lower values of fairing coverage than for a full fairing. It is a surprising finding that the fully-faired vehicle has an increasing benefit for steeper slopes on longer hills. From viewing the phase plots of these cases (not shown) it was evident that the faired vehicle benefits from being able to retain its momentum on a series of hills. The shorter the hills, the farther the vehicle is carried up the next hill by momentum. None of these conditions put any faired configuration at a disadvantage to an unfaired vehicle.

Chapter 6. Conclusions

The original purpose of this study was to determine if it is possible to find an exact solution to the governing differential equation. Due to the non-linearities of having a squared term and a derivative term in the denominator it was concluded that the chances of finding an exact solution were remote. Further effort was instead directed at solving the differential equation numerically. It was also noted during this study that any analytical solution would make it impossible to introduce real-world effects such as coasting and braking, so the time-domain evaluation is most useful anyway.

An effort was undertaken to evaluate several numerical integration methods, but these efforts were complicated by technical difficulties (getting a working C compiler). As a result, only the trapezoidal and Simpson's integration rules were evaluated (compared to a rectangular integration). The different methods resulted in a difference of less than 1% in distance travelled, so the trapezoidal method was retained because it had been used in all previous versions of the program. It was later learned that other methods would have provided better accuracy for time domain solutions. An attempt was made to evaluate more sophisticated integration techniques using Simulink/Matlab. A model was completed, but could not be verified for accuracy in the time available. It is expected, however, that since the trapezoidal integration method was used for all cases in the final study the conclusions of the study will be valid since the variations due to configurations are much larger than the integration errors.

Finally from this study it is clear that a fairing always results in an increase in speed for hill sizes up to 100m. There is no condition where the weight of the fairing (as assumed in this study for "typical" fairing construction) will cause greater slowing due to weight than the aerodynamic advantage it provides. It is also clear that the speeds reached on a faired vehicle are so much faster that better brakes are required, and better attentiveness is required from the rider due to the rapid acceleration to such speeds.

This study assumed that rider input would be constant at all times. In reality a cyclist coasts and brakes when speeds get excessive. It is proposed for future study that this program be modified to include quantified braking and coasting behavior to see if the same conclusions still hold. Hill configurations other than a constant series of identical hills should also be studied.

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